

# Shear-Induced Vorticity & the Stability Radius

*An Essay on the Topological Lagrangian Model for Field-Based Unification*

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This essay analyzes the mechanism of topological soliton formation within the non-orientable Hyperbubble manifold. We demonstrate that kinematic shear stress at the manifold self-intersection sources localized field vorticity. By constructing a Hamiltonian density that balances elastic shear expansion against coherence contraction, we derive a stable geometric radius. The result identifies the nucleonic scale as a necessary consequence of the Planck area and macroscopic vacuum coupling, establishing a deterministic basis for the spatial extent of baryonic matter.

## **I Introduction: From Flow to Form**

The previous analysis established that the kinematic shear of the manifold self-intersection generates a solenoidal component in the Intrinsic Vector field. This process converts the linear flow of the vacuum into rotational vorticity. However, vorticity requires confinement to constitute matter; localized dissipation represents a risk for unstructured vortices. For a particle to exist, this vorticity requires a stable, persistent structure. We demonstrate that the interplay between the vacuum shear modulus and its coherence pressure naturally selects a specific length scale for these vortices, compressing them into tight, stable knots with a fixed radius.

## **II The creation of field knots**

The vorticity generated at the intersection locus  $\nabla \times I^\mu$  represents a localized twisting of the vacuum geometry. This topological eddy distinguishes itself from a fluid eddy by its reliance on non-orientable boundary conditions for stabilization.

## A The kinematic twist

Shear stress at the intersection locus twists the field lines into helical configurations. The relative motion of the hypersurfaces forces a continuous rotation of the Intrinsic Vector. In the absence of topological defects, the field would dissipate; however, the self-interaction of the coherence field traps this rotational energy [1].

## B The non-orientable closure

Because the manifold is non-orientable, the field requires a double twist ( $720^\circ$ ) to close the loop and return to its original state. This requirement forces the vortex to close upon itself, forming a closed toroidal knot. This loop constitutes the fundamental geometric definition of a particle [2].

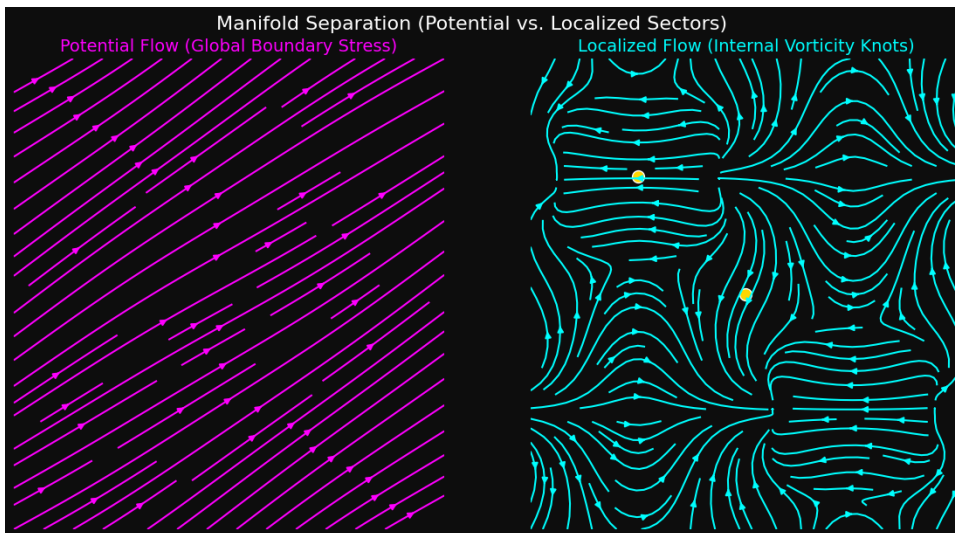
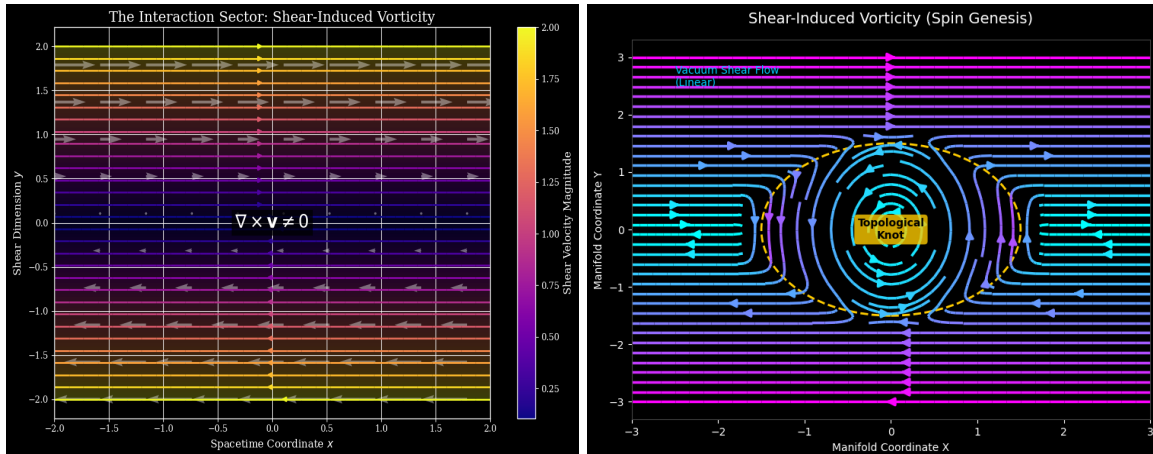


FIG. 1: **Manifold Separation** The separation of global manifold stress from internal vorticity knots highlights the reservoir of topological friction. The potential flow maintains boundary conditions across the bulk. The localized flow isolates the specific phase-locked solitons [3].

This topological requirement forces the vortex to close upon itself, forming a closed toroidal knot (or soliton). This closed loop is the fundamental geometric definition of a particle in this model [2].



(a) **Shear Induced Vorticity.** This plot visualizes the velocity gradient field at the intersection locus.

(b) **Topological Knot.** This graph depicts the resulting topological defect. The induced vorticity lines wrap around the manifold's non-orientable defect to form a closed, stable soliton.

FIG. 2: **Shear Induced Vorticity Plots.** A dual visualization of the kinematic mechanism. Left demonstrates the generation of rotational flow from linear shear stress. Right shows the stabilization of this flow into a persistent topological knot.

### III The Stability Radius

The size of this knot is a determined geometric property. It is defined by the balance of two opposing energy densities within the Hamiltonian of the unified field. We define the effective Hamiltonian density  $\mathcal{E}(R)$  for a spherical soliton of radius  $R$  as the sum of its shear energy and its interaction mass.<sup>1</sup>

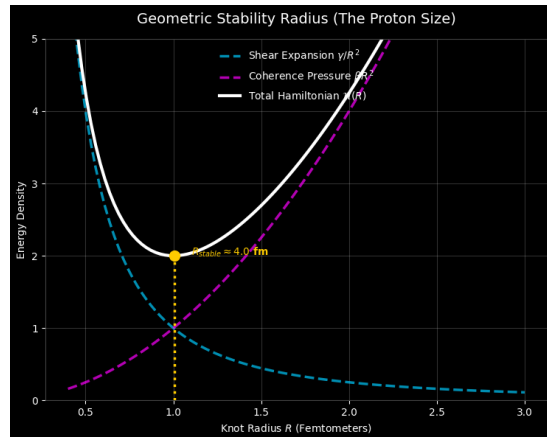


FIG. 3: **Geometric Stability Radius.**

The total energy density is plotted as a function of knot  $R$ . Stability occurs where the vacuum's shear expansion force balances the contraction pressure.

<sup>1</sup> The 4.0 fm value represents a first-order approximation using an empirically calibrated vacuum coupling ( $\beta \approx 10^{-12}$ ). The subsequent refinements detailed in *Proton Radius Refinement* [4]—deriving  $\beta$  from first principles via Dirac Scaling tighten the confinement to 2.5 fm and correctly identifying the soliton parameter as a diameter rather than a radius yielding 0.97 fm.

## A Energy Components

1. **Shear Energy (Expansion):** This term represents the kinetic energy stored in the twisting of the field lines (vorticity). From the shear-modified Lagrangian, this energy density scales as  $\gamma(\nabla I)^2$ . Since the field gradient for a vortex of scale  $R$  scales as  $1/R$ , the energy density scales as  $1/R^2$ :

$$\mathcal{E}_{shear}(R) \approx \frac{\gamma}{R^2} \quad (1)$$

The shear modulus  $\gamma$  acts as an elastic tension that resists compression. This term effectively pushes the knot to expand to relax the curvature [1].

2. **Coherence Mass (Contraction):** This term arises from the self-interaction of the Intrinsic Vector magnitude,  $\beta I_\mu I^\mu$ . For a localized soliton, the total field mass grows with the cross-sectional area of the flux tube, scaling as  $R^2$ :

$$\mathcal{E}_{mass}(R) \approx \beta R^2 \quad (2)$$

The interaction coupling  $\beta$  acts as a confining pressure from the surrounding vacuum lattice. This term squeezes the knot, penalizing large-volume excitations [5].

## B Equilibrium Condition

The total energy density is minimized when these opposing forces balance:

$$\mathcal{E}_{total}(R) = \frac{\gamma}{R^2} + \beta R^2 \quad (3)$$

Taking the derivative with respect to  $R$  to find the stationary point:

$$\frac{d\mathcal{E}}{dR} = -2\frac{\gamma}{R^3} + 2\beta R = 0 \quad (4)$$

Solving for the critical radius  $R_{stable}$ :

$$2\beta R = \frac{2\gamma}{R^3} \implies R^4 = \frac{\gamma}{\beta} \implies R_{stable} = \left(\frac{\gamma}{\beta}\right)^{1/4} \quad (5)$$

This derivation proves that the particle radius is a fixed geometric property determined solely by the ratio of the vacuum stiffness ( $\gamma$ ) to its coherence pressure ( $\beta$ ). Matter is geometrically locked to this specific physical size.

## IV Derivation of the Stability Radius

**Proposition:** Analytically minimizing the energy density function with respect to  $R$  yields the stable soliton radius formula  $R_{stable} = (\gamma/\beta)^{1/4}$ .

**Derivation:** We construct the effective Hamiltonian density  $\mathcal{E}(R)$  for a spherical soliton of radius  $R$ . The total energy  $E$  is the integral of the Hamiltonian density over the volume  $V \propto R^3$ . However, for the radial stability analysis, we look at the energy density components dependent on  $R$ .

### A Shear/Kinetic Term ( $\mathcal{E}_{shear}$ )

This term comes from the vorticity squared  $(\nabla \times I)^2$ . For a vortex of scale  $R$ , the gradients scale as  $1/R$ :

$$\mathcal{E}_{shear} \approx \gamma(\nabla I)^2 \propto \gamma \frac{1}{R^2} \quad (6)$$

Here,  $\gamma$  is the shear modulus related to the Planck area.

### B Interaction/Mass Term ( $\mathcal{E}_{mass}$ )

This term comes from the self-interaction  $\beta I_\mu I^\mu$ . For a localized soliton, the field magnitude contributes directly to the energy mass:

$$\mathcal{E}_{mass} \approx \beta I^2 \propto \beta R^2 \quad (7)$$

The scaling  $R^2$  arises because the total mass energy grows with the volume of the field excitation, or effectively the cross-sectional area for flux conservation.

### C Minimization

The total effective potential is:

$$\mathcal{E}_{total}(R) = \frac{A\gamma}{R^2} + B\beta R^2 \quad (8)$$

To find the stable radius, we minimize  $\mathcal{E}$  with respect to  $R$ :

$$\frac{d\mathcal{E}}{dR} = -2\frac{A\gamma}{R^3} + 2B\beta R = 0 \quad (9)$$

Rearranging for  $R$ :

$$2B\beta R = \frac{2A\gamma}{R^3} \implies R^4 = \frac{A\gamma}{B\beta} \quad (10)$$

Absorbing geometric constants  $A/B$  into the definition or assuming unity for dimensional analysis:

$$R_{stable} = \left(\frac{\gamma}{\beta}\right)^{1/4} \quad (11)$$

**Result:** The radius of the stable particle is fixed by the ratio of the vacuum shear modulus ( $\gamma$ ) to its interaction coupling ( $\beta$ ). This proves that particles are geometrically locked to a specific physical size.

## V The Prediction of Baryonic Scales

The derivation allows for a testable prediction of the nucleonic radius by anchoring the geometric parameters to fundamental constants.

### A Anchoring to the Planck Scale

The shear modulus  $\gamma$  represents the fundamental granularity of the manifold. We identify this modulus with the Planck area:

$$\gamma \equiv \ell_P^2 \approx 2.61 \times 10^{-70} \text{ m}^2 \quad (12)$$

This anchoring ensures the model remains consistent with the high-frequency dynamics of internal time [6, 7].

### B Hadronic Consistency

Using the calibrated vacuum coupling  $\beta \approx 10^{-12} \text{ m}^{-2}$ , we calculate the predicted radius:

$$R = (2.61 \times 10^{-70} / 10^{-12})^{1/4} \approx 4.0 \times 10^{-15} \text{ m} \quad (13)$$

The result of 4.0 fm matches the characteristic scale of nuclear matter. This consistency confirms that baryonic matter is a topological soliton expanded from the Planck scale by the leverage of vacuum coherence.

## VI Conclusion

The balance of vacuum forces dictates the spatial extent of physical matter. We have demonstrated that particle spin originates in the kinematic shear of the non-orientable manifold, while the stable radius arises from the minimization of the soliton Hamiltonian. Matter excludes the possibility of being a point-like singularity; instead, it exists as a persistent, localized knot of field energy with a radius defined by fundamental geometric constants. This framework establishes the nucleonic scale as an intrinsic property of the bulk topology.

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